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Enhancing the Power System Observability with the Aid of Phasor Measurement Units

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Abstract

This paper presents an approach for optimally allocating the Phasor Measurement Units (PMUs) in a power system. The proposed approach is based on Binary Integer Programming (BIP) to minimize the total number of PMUs that can achieve full system observability. The full system observability is ensured both in normal operating conditions and in case ofN-1 contingencies such as the outage of a PMU or a transmission line. Moreover, the approach is utilized to allocate the PMUs in case of limited number of PMU channels. The problem formulation considers the Zero Injection Buses (ZIBs) and uses a set of rules that can improve the redundancy of the PMUs by choosing better locations without increasing their number. The proposed approach is applied on the IEEE standard systems 14, 30 and 57 test systems. The simulations results are compared with other approaches used in the literature to validate the performance of the proposed approach.

Index Terms

Binary Integer programming, observability, optimal placement, phasor measurement units, channel limitation

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1 Introduction

The phasor measurement of the bus voltages and line currents is of great importance in the analysis and operation of the power system. However, to get accurate system observability, all measurements should be synchronized based on the same GPS time reference using phasor measurement units (PMUs). [1]

Full observability of a power system can be achieved by placing a limited number of PMUs on carefully selected buses to achieve the full system observability at minimum cost. This requires formulating an optimization problem to find the minimum number and location of PMUs that can achieve this task. In addition, the PMU placement should be robust enough to maintain the system observability under N-1contingencies such as PMU failure or a transmission line outage[2]. The assumption of unlimited number of measuring channels of a PMU is not always valid in practice as it might increase the cost of the system without any benefit. Thus, it is also important to identify the minimum number of channels that can achieve the full system observability.

The PMU placement problem has been addressed in several studies. For example, in[2], a binary particle swarm optimization algorithm was used for minimization of the number of PMUs and the maximization of the measurement redundancy. However, the study didn't consider the limitation of the measuring channels or the contingency conditions. In[3], a modified binary particle swarm optimization algorithm was used to find the optimum location for the PMUs under normal and

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contingency conditions. However, the study didn't consider the case of limited PMU channels. In [4], integer quadratic programming approach is used to minimize number of PMUs required in normal conditions and in case of PMU loss only without considering PMU channel limitation. In [5], integer linear programming approach is used to minimize number of PMUs in normal conditions and contingency. Also number of PMU channels considered. In[6]integer linear programming approach is used to determine the optimum location of PMUs that can lead to full system observable under normal and contingency conditions. However, the limited measuring channels in the PMU not considered. In [7] and [8] a binary integer programming was used for PMU placement taking channel limitation into account. However, the study considered the loss of PMU only in the contingency analysis. In[9],the proposed method provides suitable constraints for power systems with two adjacent injection measurements (IMs) to minimize number of PMUs and maximize redundancy without considering channel limitation and without studying any contingency conditions. In[10],the optimal PMU placement was studied by applying binary integer programming method, considering channel limitation in normal condition and in contingency conditions, but the simulation results didn't make the system fully observable due to issues in the constraints of the optimization problem.

The aim of this paper is to identify the optimum number and locations of PMUs for full system observability under normal and contingency conditions while considering the limitation in the PMU channels.

2 PMU Placement Under Normal Conditions

The objective of the PMU placement problem is to determine the minimum number of PMUs for full network observability. The general PMU placement problem can be formulated and solved as a Binary Integer Programming problem as follows:

$$\text{Minimize} \quad \sum_{i=1}^{N} w_i\, x_i \qquad (1)$$

$$\text{Subjected to } \boldsymbol{f} \geq \boldsymbol{U} \qquad (2)$$

where N is the number of system buses, w_i represents the installation cost of the PMUs, x_i is a binary variable,$\boldsymbol{f}$is$N\times 1$observability function vector with its i^{th} entry representing the observability of bus i and $\boldsymbol{U}$ is an $N\times 1$ unity vector. The vector $\boldsymbol{f}$ can be given by $\boldsymbol{f} = \boldsymbol{A}\,\boldsymbol{X}$ where $\boldsymbol{A}$ is the $N\times N$ connectivity matrix with its entries defined as:

$$a_{ij} = \begin{cases} 1, & if \text{ i} = \text{j} \\ 1, & if \text{ nodes } i \text{ and } j \text{ are connected} \\ 0, & \text{otherwise} \end{cases} \qquad (3)$$

$\boldsymbol{X}$ is an$N\times 1$ binary decision variable vector with its entries are x_i which can be defined as:

$$x_{\text{i}} = \begin{cases} 1, & \text{if node } i \text{ has a PMU,} \\ 0, & \text{Otherwise} \end{cases} \quad i = 1, \dots, N \qquad (4)$$

The non-zero entries in $\boldsymbol{X}$ indicates the buses which can be observed by a PMU located at a certain bus.

According to the topological observability analysis, the following rules are used to facilitate the formulation of the optimization problem:

Rule 1: A PMU that is installed at a given bus will make this bus and all incident buses to it observable. This rule is interpreted in equations (2) and (4) which can be explained for Bus 1 and Bus 2 of the 7-bus system shown in Fig. 1 as follows[3],[6]:

$$f_1 = x_1 \mid x_2 \;\; \geq 1 \qquad (5)$$

$$f_2 = x_1 \mid x_2 \mid x_3 \mid x_6 \mid x_7 \;\; \geq \qquad (6)$$

In these inequalities, the operator " | " serves as the logical "OR" and the use of 1 in the right hand side of the inequality ensures that at least one of the variables appearing in the sum will be non-zero, and thus, Bus i becomes observable.

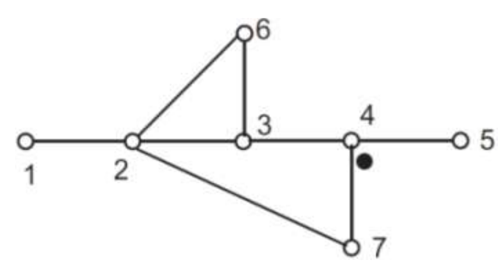

Fig.(1) 7-Bus System

Rule 2: If a set of M buses are connected through a ZIB, then all these buses will be observable if only M-1 buses are observable. This can be achieved by applying Kirchhoff's current law (KCL) at the ZIB. This rule can be implemented using the following inequality[3],[6]:

$$\sum_{i=1}^{M} f_i \geq M-1 \tag{7}$$

In Fig.(1),Bus (4) is a ZIB so the inequality will be:

$$f3 |\ f4 \ |\ f5 \ |\ f7 \geq\ 3 \tag{8}$$

Rule 3: For a set of buses connected through two adjacent ZIBs, the inequality for one of the ZIBs should follow Rule (2). The second inequality should consider all the remaining buses in the set. This rule can be extended in the same manner for more than two adjacent ZIBs. For the system in Fig. (1), Bus (3) and Bus (4) are two adjacent ZIBs, thus, the inequalities for these buses should be written as follows[6]:

$$f2 \ |\ f3 \ |\ f4 \ |\ f6 \ \geq\ 3, \tag{9}$$

$$f5 \ \ |\ f7 \geq\ \ 1, \tag{10}$$

Rule 4: If there are two ZIBs connected to each other through another bus such that inequality (8) is applied on each ZIB, then their sum can be considered as a single constraint. For the system in Fig. (1), if Bus (2) and Bus (4) are ZIBs connected together through Bus (3), then the constraint in this case is[6]:

$$f1 \ |\ f2 \ \ |f3 \ \ |\ f4 \ |f5 \ |\ f6 \ |f7 \ \geq\ 5 \tag{11}$$

Rule 5:If a set of buses are connected to a ZIB where Bus *i* is connected directly to the ZIB and also connected to it through another adjacent bus, Rule (2) must be applied excluding Bus *i* which should follow Rule (1). This is to avoid the unobservability of the system if only Rule (2) is applied to the whole set. For the system in Fig. (1),let's consider that Bus (2) is a ZIB and Bus (6) is connected to it directly and also through Bus (3), so the inequality will be[6]:

$$f1 |\ f2 \ |\ f3 \ |\ f7 \ \geq\ 3 \tag{12}$$

$$f6 \ \geq\ 1 \tag{13}$$

3 PMU Placement with Limited Number of Channels Under Normal Conditions

Consider a PMU which has L channels and installed at bus k, also assume that bus k is connected to N_k number of neighbor buses. If the number of channels, L, is larger than the number of neighbor buses, N_k, then a single PMU placed at the bus will provide phasor voltages at all its neighbor buses. Otherwise, there will be r_k combinations of possible channel assignments to branches incident at bus $k$as follows[7]:

$$r_k = \begin{cases} {}^{N_k}C_l & if\ N_k > L \\ 1 & if\ N_k \leq L \end{cases} \tag{14}$$

where the number of possible combinations of L out of N_k branches is defined as:

$${}^{N_k}C_l = \frac{N_k!}{(N_k - L)!\,L!} \tag{15}$$

To consider the limited number of channels, a new $S{\times}N$ connectivity matrix, $\boldsymbol{A^{new}}$ is formed using the original connectivity matrix $\boldsymbol{A}$ where $S = \sum_1^N r_k$. For each bus, if r_k = 1, then the row in $\boldsymbol{A^{new}}$ associated with this bus is the same as that in $\boldsymbol{A}$. If r_k> 1, then there will be ${}^{N_k}C_l$rows associated with this bus. For the system shown in Fig. 1, if the PMUs have 2 channels, the matrix $\boldsymbol{A^{new}}$ is:

$$\boldsymbol{A^{new}} = \left[\begin{array}{c} 1\,1\,0\,0\,0\,0\,0 \\ \hline 1\,1\,1\,0\,0\,0\,0 \\ 1\,1\,0\,0\,0\,1\,0 \\ 1\,1\,0\,0\,0\,0\,1 \\ 0\,1\,1\,0\,0\,1\,0 \\ 0\,1\,1\,0\,0\,0\,1 \\ 0\,1\,0\,0\,0\,1\,1 \\ \hline 0\,1\,1\,1\,0\,0\,0 \\ 0\,1\,1\,0\,0\,1\,0 \\ 0\,0\,1\,1\,0\,1\,0 \\ \hline 0\,0\,1\,1\,1\,0\,0 \\ 0\,0\,1\,1\,0\,0\,1 \\ 0\,0\,0\,1\,1\,0\,1 \\ \hline 0\,0\,0\,1\,1\,0\,0 \\ \hline 0\,1\,1\,0\,0\,1\,0 \\ \hline 0\,1\,0\,1\,0\,0\,1 \end{array}\right] \begin{array}{l} Bus\ 1 \\ \\ \\ \\ Bus\ 2 \\ \\ \\ \\ \\ Bus\ 3 \\ \\ \\ Bus\ 4 \\ \\ Bus\ 5 \\ Bus\ 6 \\ Bus\ 7 \end{array}$$

Based on the new connectivity matrix $\boldsymbol{A^{new}}$, the PMU placement problem can be formulated as follows:

$$\min \sum_{i=1}^{N} w_i y_i \tag{16}$$

Subject to:$\boldsymbol{F} \geq \boldsymbol{U}$ (17)

where y_i is a binary variable,$\boldsymbol{F}$is$N{\times}1$observability function vector with its i^{th} entry representing the observability of bus *i* and can be given by:

$$\boldsymbol{F} = \ (\boldsymbol{A^{new}})^T \boldsymbol{Y} \tag{18}$$

where$\boldsymbol{Y}$is$S{\times}1$decision variable vector matrix with its i^{th} entry is y_i. The non-zero entries in $\boldsymbol{Y}$ indicate the buses which can be observed by a PMU located at a certain bus. Also, Rules (1) to (5) should be considered as before.

For the system of Fig.(1) buses 1,2,6 are not incident to any ZIBs so the constraints will be as follow:

$$F_1 = y_1 \mid y_2 \mid y_3 \mid y_4 \geq 1 \quad (19)$$

$$F_2 = y_1 \mid y_2 \mid y_3 \mid y_4 \mid y_5 \mid y_6 \mid y_7 \mid y_8 \mid y_9 \mid y_{15} \mid y_{16} \geq 1 \quad (20)$$

$$F_6 = y_3 \mid y_5 \mid y_7 \mid y_9 \mid y_{10} \mid y_{15} \geq 1 \quad (21)$$

Also, as the buses 3, 5 and 7 are incident to the ZIB, so the constraints will be as follows:

$$F_3 \mid F_4 \mid F_5 \mid F_7 = y_2 \mid y_4 \mid 2y_6 \mid y_7 \mid 2y_8 \mid y_9 \mid 2y_{10} \mid 3y_{11} \mid 3y_{12} \mid 3y_{13} \mid 2y_{14} \mid y_{15} \mid 2y_{16} \geq 3 \quad (22)$$

In this case, the solution of the PMU placement problem for a specified channel limits for PMUs is 2, will be given as follow:

$$Y = [\,0\;0\;0\;1\;0\;0\;0\;0\;0\;1\;0\;0\;0\;0\;0\;0\,]^T \quad (23)$$

This yields a total of 2 PMUs, one is placed at bus 2, and another one at bus 3.

4PMU Placement with Limited Number of Channels Under N-1 Contingency

To guarantee the full observability of the power system when a single PMU loss occurs, each bus must be observed by at least two PMUs. Hence, the elements in the vector ***U*** in Equation (17) are multiplied by 2 [5],[6],[7].The same concept is applied to guarantee the full observability of the system when a single line outage occurs. However, for any bus connected to the system via a radial feeder, its corresponding entry in ***U*** is kept unity. This is because the loss of the radial feeder leads to the disconnection of the bus connected to it, and hence, it is not needed to be observable.

4 Simulation Results

To evaluate the ability of the presented formulation, the 14-Bus, 30-Bus and 57-Bus IEEE standard systems are used to identify the optimum locations of the PMUs under normal conditions and in case of a single line outage and a single PMU loss.

The results of the presented formulation for different cases are displayed in Table 2 to Table 4 while the comparison with the results of the previous studies in the literature[5],[7],[10] is presented in Table 5.

An interesting observation from the results in Table 2 to Table 4 is that for all three systems the maximum number of needed channels is only 4 channels. Obtaining PMUs with more channels will only increase the cost with no benefit with regards to the observability.

It can be observed from the comparison in Table 5 that under normal conditions for almost all the considered systems, the number of PMUs obtained using the presented formulation is either equal to or lower than those of obtained from the previous studies. However, for the IEEE 30-Bus system 7 PMUs with 4 channels are required for full system observability which is more than the 6 PMUs in[10]. By observing the locations of the PMUs given in[10], it can be seen that Bus 29 and Bus 30 are not observable. This is because the rules considered for modeling the ZIBs are not applicable in all cases. For contingency conditions, the number of PMUs obtained using the presented formulation is either equal to or more than those obtained from the studies in [5], [7] and[10]. However, it should be noted that these studies did not show the locations for the PMUs under contingency conditions, and hence, the full observability of the system cannot be verified under these conditions.

5 Conclusion

This paper presents an efficient and comprehensive formulation based on binary integer programming method for the optimal placement of PMUs. The formulation took into account the availability of number of channels under normal conditions and N-1 contingencies, including single line outage and single PMU loss. The proposed formulation was tested on three IEEE standard systems and the results were compared with previous studies. The comparison showed that the presented formulation is capable of finding the optimum locations PMU placement for full network observability with the availability of limited channels of PMUs. Also with limited channels more than 4 channels will increase the cost only regarding to the system observability.

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Table (1). IEEE standard system [6].

IEEE system	No. of lines	No. of ZIBs	Location of ZIBs	No. of radial buses	Location of radial buses	Max. No. of lines connected to a bus
14 Bus	20	1	7	1	8	5
30 Bus	41	6	6,9,22,25,27,28	3	11,13,26	7
57 Bus	80	15	4,7,11,21,22,24,26, 34,36,37,39,40,45,46,48	1	33	6

Table (2).IEEE 14-bus system optimal PMU placement in different cases

14-BUS SYSTEM PMU PLACEMENT									
System state	Normal			PMU outage			Line Outage		
No. of channels	No. of PMUs	Location	PMUs Buses	No. of channels	Location	PMUs Buses	No. of channels	Location	PMUs Buses
1	7	1,4,6,7,10,12,13	0.50	13	2,2,4,5,6,6,7,7,9,10,11,13,14	0.93	13	2,2,4,5,6,6,7,7,9,10,11,13,14	0.93
2	5	1,2,6,9,14	0.36	9	2,2,3,5,6,9,10,13,14	0.64	9	2,2,3,5,6,9,10,13,14	0.64
3	4	2,6,7,9	0.29	7	2,4,5,6,9,10,13	0.50	7	2,4,5,6,9,10,13	0.50
≥4	3	2,6,9	0.21	7	2,4,5,6,9,10,13	0.50	7	2,4,5,6,9,10,13	0.50

Table (3).IEEE 30-bus system optimal PMU placement in different cases

30-BUS SYSTEM PMU PLACEMENT									
System state	Normal			PMU outage			Line Outage		
No. of channels	No. of PMUs	Location	PMUs Buses	No. of PMUs	Location	PMUs Buses	No. of PMUs	Location	PMUs Buses
1	12	1,2,2,4,10,13,15,17, 19,24,24,27	0.40	24	1,3,5,10,12,13,14, 15,16,17,18, 19,20,22,23,27,29	0.80	23	2,3,4,5,7,9,12,13,15,15,16, 17,19,20,20,23,24,24,27,28, 28,29,29	0.77
2	8	3,7,9,10,12,15,19,27	0.27	16	2,2,3,4,10,10,12,12, 15,1923,29,30	0.53	15	1,3,5,7,10,12,15,16, 17,18,19,24,24,27,27	0.50
3	7	2,4,10,12,15,18,27	0.23	14	1,3,5,7,10,12,14,15 ,17,19,20,24,29,30	0.47	13	1,3,5,7,10,12,15, 17,19,20,24,29,30	0.43
≥4	7	2,4,10,12,15,18,27	0.23	14	1,3,5,7,10,12,14,15, 17,19,20,24,29,30	0.47	13	1,3,5,7,10,12,15, 17,19,20,24,29,30	0.43

Table (4). IEEE 57-bus system optimal PMU placement in different cases

57-BUS SYSTEM PMU PLACEMENT									
System state	Normal			PMU outage			Line Outage		
No. of channels	No. of PMUs	Location	PMUs Buses	No. of PMUs	Location	PMUs Buses	No. of PMUs	Location	PMUs Buses
1	21	1,4,8,12,13,20,28,30, 32,33,38,51,53,55,56	0.37	42	1,1,2,3,3,3,4,4,9,9,14,16, 17,18,19,26,27,27,28,29, 30,31,32,32,33,38,42,42, 43,43,45,48,48,49,49,49, 50,51,51,53,54,54	0.74	42	1,1,2,3,3,3,4,4,9,9,14,16, 17,18,19,26,27,27,28,29, 30,31,32,32,33,38,42,42, 43,43,45,48,48,49,49,49, 50,51,51,53,54,54	0.74
2	14	1,4,4,9,12,13,20,27,31, 32,34,41,50,53	0.24	28	1,1,3,4,4,4,9,10,12,13,13, 13,14,14,19,20,25,27,29, 30,32,32,49,51,53,54,56	0.49	28	1,1,3,4,4,9,10,1,1,12,13, 14,15,18,20,22,25,27,29, 31,32,34,49,51,53,54,56,56	0.49
3	12	1,3,9,10,18,24,25, 29,32,49,54,56	0.21	24	1,1,4,9,12,15,18,20,25,27, 29,30,32,33,37,38,38,41, 46, 50,51,53,54,56	0.42	23	1,1,4,10,12,13,15,18, 19,24,25,28,29,31,32, 37,41,49,50,52,54,55,56	0.4
≥4	11	1,4,13,20,25,29,32, 36,51,54,56	0.19	22	1,2,4,9,12,15,18,20,25, 27,29,30,32,33,34,38,42, 50,51,53,54,56	0.38	21	1,3,4,9,12,15,19,20,25, 27,29,31,32,34,38,42,50, 51,53,54,56	0.37

Table (5).Comparison between proposed method and different methods in the literature

Method	No. of Channels	IEEE 14-bus			IEEE 30-bus			IEEE 57-bus		
		Normal	*PMU outage*	*Line Outage*	*Normal*	*PMU outage*	*Line outage*	*Normal*	*PMU outage*	*Line outage*
Proposed method	1	7	13	13	12	24	23	21	42	42
Ref.[5]		7	-	-	12	-	-	21	-	-
Ref.[7]		7	12	-	14	23	-	23	34	-
Ref.[10]		7	12	-	12	21	-	21	34	-
Proposed method	2	5	9	9	8	16	15	14	28	28
Ref.[5]		5	-	-	8	-	-	14	-	-
Ref.[7]		5	8	-	9	17	-	14	23	-
Ref.[10]		5	8	-	8	14	-	14	23	-

Proposed method	3	4	7	7	7	14	13	12	24	23
Ref.[5]		4	-	-	7	-	-	12	-	-
Ref.[7]		4	7	-	8	14	-	12	22	-
Ref.[10]		4	7	-	7	13	-	12	22	-
Proposed method	≥ 4	3	7	7	7	14	13	11	22	21
Ref.[5]		3	7	7	7	15	13	11	26	19
Ref.[7]		4	7	-	7	15	-	11	22	-
Ref.[10]		3	7	-	6	12	-	11	22	-